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PROPERTIES OF THE SQUARE-ROOT DERIVATIVE TYPE CURVE

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Abstract

The conventional way to analyse well test data is by a combination of type curve analysis and special analysis for specific flow periods. In many cases, these flow periods, radial, linear, spherical, etc., are identified by inspection of type curves. Once identified, the pressure versus some time function may be plotted as a straight line. Then, reservoir and well properties may be computed from the slope of the line and its position in the coordinate system. In this work, we propose a new type curve based on derivatives with respect to the square-root of time. The resulting type curve is useful for identifying linear flow regimes. Linear flow periods may appear in many cases, for example: flow towards horizontal wells, vertically fractured wells, etc.

1. Introductory remarks

The square-root derivative type curve was developed as a result of our investigations on one phase flow towards restricted entry horizontal wells. Hence, the properties of the square-root derivative type will be discussed in this context.

A horizontal well may be defined as a well that is drilled parallel to the reservoir bedding plane. Developments in drilling technology have made horizontal drilling a practical alternative to conventional drilling. The insignificant pressure drop observed in horizontal wells indicates infinite conductivity flow conditions in the wellbore. It has been shown, however, that a uniform flux solution evaluated at the equivalent pressure point accurately mimics the behaviour of the corresponding infinite conductivity solution [9, 10].

From a mathematical point of view, there is no difference between restricted entry vertical wells and horizontal wells. Since most producing formations are thin compared to their areal extent, the interaction between the well and the exterior boundaries will be different for the two cases.

Previous investigations have shown that the pressure response of a constant flux horizontal well in a box-shaped reservoir may exhibit four distinct flow periods [1]. These flow periods are:

(1) Early radial flow period.
(2) Early linear flow period.
(3) Late pseudo radial flow period.
(4) Late linear radial flow period.

Since the flow periods are consequences of the interaction between the well and the exterior boundaries, it is quite possible that one or more of these may not appear in an actual test. Then it may be advantageous to have quick methods to verify the existence of those flow regimes that may be observed in a given test.

In these distinct flow periods, the complex mathematical solution may be approximated by simpler expressions, referred to as limiting forms. These usually plot as straight lines and as a consequence have very simple interpretations. There is a possibility, however, that one may assume such a period when it actually does not happen. Then the resulting analysis will be erroneous. Hence, the correct identification of such a period is critical.

While Odeh and Babu [1] proposed time criteria, we will propose the use of derivative type curves. The latter approach can be used for cases where the field data are not too contaminated by noise.

2. Properties of the square-root derivative type curve

We consider the flow of a slightly compressible fluid of constant viscosity towards a horizontal well of length L_w, arbitrarily located in a box-shaped reservoir. The reservoir dimensions are L_x, L_y and L_z, respectively. The well axis is parallel to the y-axis and the reservoir permeability is anisotropical. Listed in appendix A are the corresponding differential equations, and the dimensionless variables are given in appendix B.

Line source solutions for uniform flux horizontal wells in slab- or box-shaped reservoirs are well known [1–5]. For a box-shaped reservoir, the solution (to eqs. (A.1)–(A.3)) is:

$$p_d(x_d, y_d, z_d, t_d) = \pi L_{zd} \int_0^{t_d} \frac{\Delta p_{sx}}{\mathrm{d}x'_d} \left[\int_{y_{1d}}^{y_{2d}} \Delta p_{sy} \right] \frac{\Delta p_{sz}}{\mathrm{d}z'_d} \, \mathrm{d}\tau. \tag{1}$$

Here, we have assumed that production is at a constant rate and that the well may be modelled by a line source.

The instantaneous "point" source solution for flow to the point (x'_d, y'_d, z'_d) is given by the following [6]:

$$\Delta p_s(t) = \Delta p_{sx} \times \Delta p_{sy} \times \Delta p_{sz} \,, \tag{2}$$

where Δp denotes pressure drop and the point of interest is given by the coordinates x_d, y_d, and z_d. The terms in the source function Δp_s are listed in appendix C. The details regarding the computational procedure for evaluation of eq. (1) have been discussed in a companion paper [7].

Let p'_{dw} denote the derivative of the dimensionless wellbore pressure with respect to dimensionless time t_d. Then the general equation for the square-root derivative is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_d}} = 2\sqrt{t_d}\, p'_{dw} \tag{3}$$

and the logarthmic derivative [8]:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\ln t_d} = t_d p'_{dw}\,. \tag{4}$$

The above derivatives have some interesting special cases. In the case of radial flow, the corresponding limiting form for the dimensionless pressure is:

$$p_{dw} = A \ln t_d + B\,, \tag{5}$$

where A and B are constants. The value of the constants depends on whether the flow is of the late pseudo radial or of the early radial type (cf. appendix D). Then the square-root derivative is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_d}} = 2A\, t_d^{-1/2} \tag{6}$$

and the logarithmic derivative:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\ln t_d} = A\,. \tag{7}$$

Let m_{sq} and m_l denote the slope of the square-root and logarithmic derivatives, respectively, when plotted on log–log paper.

Note that the square-root derivative, eq. (6), will plot as a straight line with slope $m_{sq} = -1/2$ on log–log paper in a radial flow period. The logarithmic derivative, on the other hand, will plot as a straight line at a constant value, that is, $m_l = 0$. Then it is easy to identify radial flow from the logarithmic derivative type curve.

For both derivatives, the effect of the constant A is to shift the straight line vertically in the coordinate system.

In the case of a linear flow period, eq. (1) will have the following asymptotical form:

$$p_{dw} = A\, t_d^{1/2} + B\,, \tag{8}$$

where the value of the constants A and B depends on whether the linear flow regime is of the early or late type (cf. appendix D). Taking the square-root derivative of eq. (8) yields:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_{dw}}} = A. \tag{9}$$

The logarithmic derivative of eq. (8) is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\ln t_d} = \tfrac{1}{2} A\, t_d^{1/2}. \tag{10}$$

Note that the square-root derivative, eq. (9), will plot as a straight line at a constant value during the linear flow periods. Hence, this flow regime is very easy to identify using the square-root derivative type curve. The logarithmic derivative, on the other hand, will plot as a straight line with slope $m_{sq} = 1/2$ on log–log paper.

During the pseudo-steady state period and during the early period, when the pressure response is dominated by wellbore storage, it is commonly accepted that the pressure is a linear function of time. Then,

$$p_{dw} = A\, t_d + B, \tag{11}$$

where the value of the constants A and B depends on the flow regime. Taking the square-root derivative of eq. (11) yields:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_{dw}}} = 2A\, t_d^{1/2} \tag{12}$$

and the logarithmic derivative is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\ln t_d} = A\, t_d. \tag{13}$$

Observe that both derivatives will plot as straight lines on log–log paper. The square-root derivative straight line will have slope $m_{sq} = 1/2$, while the logarithmic derivative will have a slope $m_l = 1$.

During the bilinear flow period, for a finite conductivity fracture, the pressure response is given by:

$$p_{dw} = A\, t_d^{1/4} + B. \tag{14}$$

Then the square-root derivative of eq. (14) is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_{dw}}} = \tfrac{1}{2} A\, t_d^{-1/4} \tag{15}$$

and the logarithmic derivative is:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\ln t_d} = \tfrac{1}{4} A\, t_d^{1/4}. \tag{16}$$

Note that the two derivative type curves will plot as straight lines with slope $m_{sq} = -1/4$ and $m_l = 1/4$, respectively.

3. The fourth-root derivative

In the same way as we proposed a square-root derivative for easy identification of linear flow, one may propose a fourth-root derivative for easy identification of a bilinear flow period.

Then this derivative may be computed from the chain rule as:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}t_d^{1/4}} = 4\, t_d^{3/4} p_d'. \tag{17}$$

Taking the derivative of eq. (14) with respect to the fourth-root of t_d yields:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}t_d^{1/4}} = A. \tag{18}$$

Hence, the fourth-root derivative type curve, eq. (18), will plot as a horizontal straight line during the bilinear flow period.

4. Test on the square-root derivative

The first test case is for a well/reservoir system where the length of the well is small compared to the reservoir dimensions. This corresponds to a drainhole. The result is plotted in fig. 1. Here, we have used the conventional approach, i.e. plotting dimensionless pressure and the logarithmic derivative in log–log coordinates. In this case, two radial flow periods are easily identified by the horizontal lines in the logarithmic derivative. The interpretation of the transitional period between the two radial periods is not so obvious.

The transitional curve may look like a straight line. Hence, there is a possibility for linear flow. If it can be shown that the slope $m_{sq} = 1/2$, then one may conclude

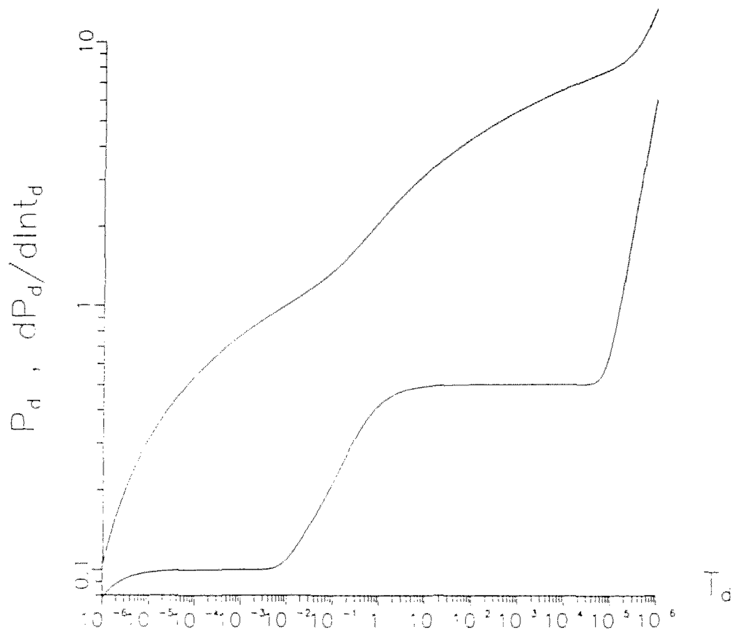

Fig. 1. Well located at $x_d = 500$, $500 < y_d < 502$, $z_d = 0.25$. Reservoir dimensions: $L_{xd} = 1000$, $L_{yd} = 1002$, $L_{zd} = 0.4$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

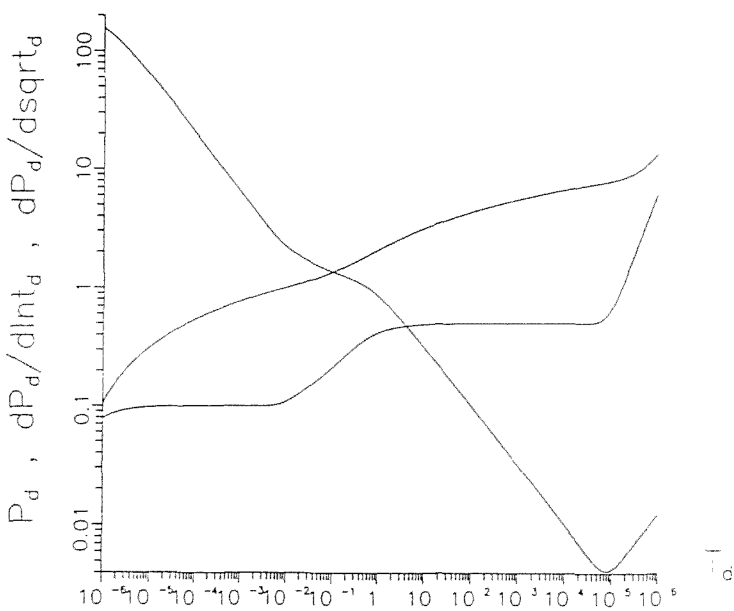

Fig. 2. Well located at $x_d = 500$, $500 < y_d < 502$, $z_d = 0.25$. Reservoir dimensions: $L_{xd} = 1000$, $L_{yd} = 1002$, $L_{zd} = 0.4$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

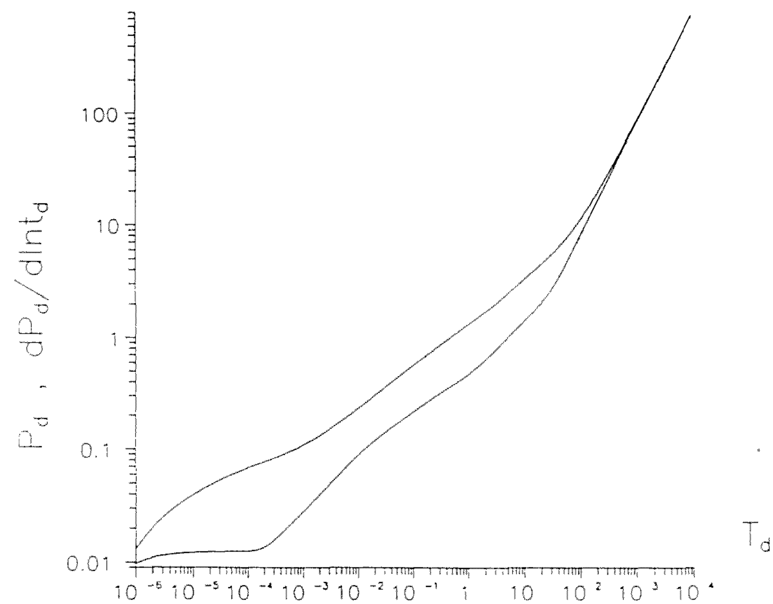

Fig. 3. Well located at $x_d = 10$, $1 < y_d < 3$, $z_d = 0.025$. Reservoir dimensions: $L_{xd} = 20$, $L_{yd} = 4$, $L_{zd} = 0.05$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

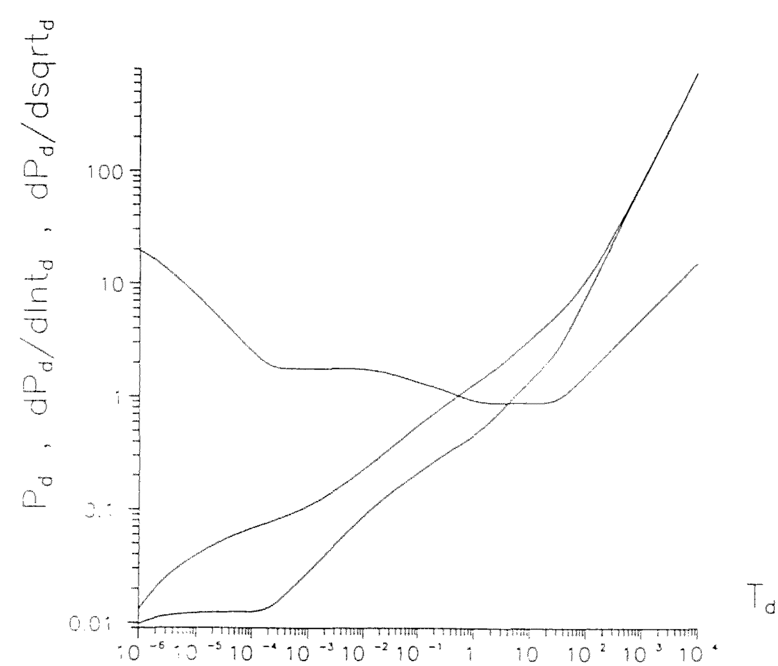

Fig. 4. Well located at $x_d = 10$, $1 < y_d < 3$, $z_d = 0.025$. Reservoir dimensions: $L_{xd} = 20$, $L_{yd} = 4$, $L_{zd} = 0.05$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

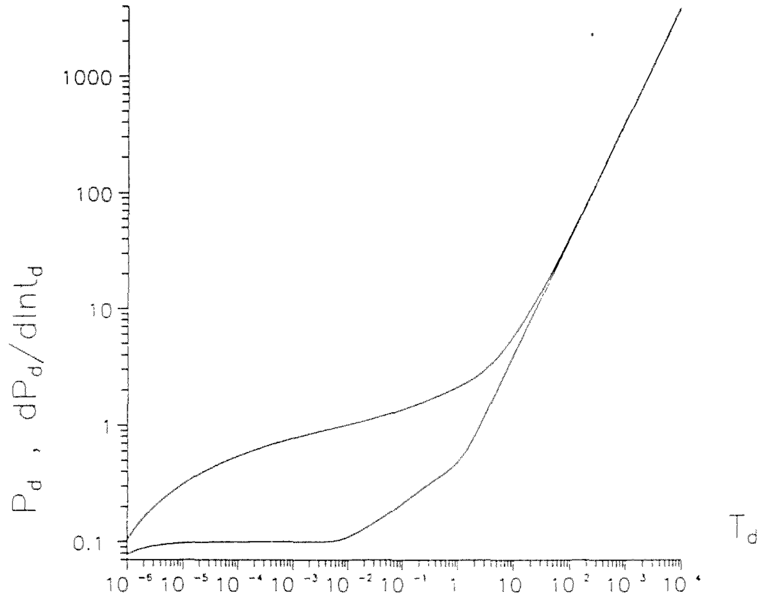

Fig. 5. Well located at $x_d = 2$, $1 < y_d < 3$, $z_d = 0.25$. Reservoir dimensions: $L_{xd} = 4$, $L_{yd} = 4$, $L_{zd} = 0.4$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

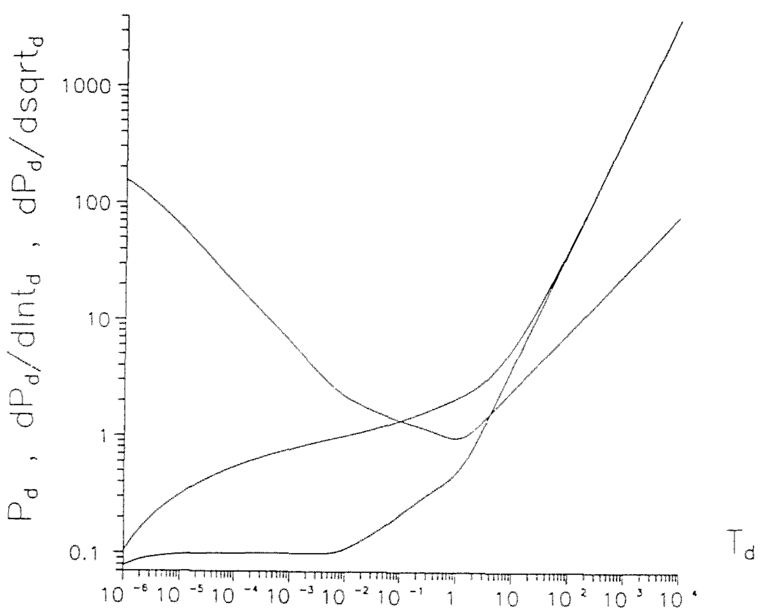

Fig. 6. Well located at $x_d = 2$, $1 < y_d < 3$, $z_d = 0.25$. Reservoir dimensions: $L_{xd} = 4$, $L_{yd} = 4$, $L_{zd} = 0.4$. $k_x/k_y = 1$, $k_z/k_x = 1$, $r_{wd} = 0.001$.

that the flow is of the linear type. To reach this conclusion requires computations. The more direct approach is to plot the square-root derivative. This has been done in fig. 2. Using this approach, a linear flow period may be identified by a horizontal line in the square-root derivative type curve. Observe that no such line exists. Hence, one may conclude that the linear flow period does not exist for this well/reservoir configuration. Furthermore, observe that the radial flow periods plot as straight lines with slope $m_l = -1/2$. In addition, observe the transition between the late radial flow period and the pseudo-steady state period where the square-root derivative changes from a straight line with slope $m_l = -1/2$ to another one with slope $m_l = 1/2$.

The next test case is for a rectangularly shaped drainage area, where the length of the well is of the same order of magnitude as the drainage area. In addition, the height of the reservoir is small compared to the length of the well. The pressure response, using the conventional type curves is shown in fig. 3. By inspection of the pressure derivative type curve, one may conclude that an early radial flow period exists. In the next stage one may see a possible straight line. Then the curve exhibits some curvature and then once again a possible straight line section before going into pseudo-steady state. Hence, in this case there is a possibility of two linear flow periods. Again, we have plotted the result using the square-root derivative, cf. fig. 4. Note that by using this plot, it is easy to identify two linear flow periods. The last one corresponds to flow in a sand channel.

As a final example, we tested a well/reservoir system characterized by a square drainage area and a well length that is of the same order of magnitude as the length of the reservoir. In this case, however, the reservoir height L_{zd} is larger than for the previous test case. The result using the conventional type curves is shown in fig. 5. From this plot, we may conclude that there is a radial flow period initially. Then there is a possible straight line section in the logartithmic derivative before pseudo-steady state is reached. Hence, a linear flow period is possible. From the square-root derivative type curve, shown in fig. 6, one may conclude that there is no linear flow period in this case. It is interesting to note that for this sytem, there is an initial radial flow period, a transitional period and then pseudo-steady state only.

5. Conclusions

A new type curve has been proposed. When combined with the logarithmic derivative type curve, it is useful for easy identification of flow regimes.

This type curve is easily incorporated into an existing program. It requires one additional line of coding only:

$$\frac{\mathrm{d}p_{dw}}{\mathrm{d}\sqrt{t_d}} = 2\sqrt{t_d}\,p'_{dw} . \tag{19}$$

One normally computes the time derivative of pressure anyway to compute the logarithmic derivative.

We believe that the main application for the new type curve will be for the horizontal wells, vertically fractured wells, and channel sands.

Nomenclature

c_t = system compressibility (psi^{-1}).
h = reservoir height. Same as L_z (ft).
k_x = absolute reservoir permeability in the x-direction (md).
k_y = absolute reservoir permeability in the y-direction (md).
k_z = absolute reservoir permeability in the z-direction (md).
L_w = well length (ft).
L_x = reservoir length in the x-direction (ft).
L_{xd} = dimensionless reservoir length in the x-direction.
L_y = reservoir length in the y-direction (ft).
L_{yd} = dimensionless reservoir length in the y-direction.
L_z = reservoir length in the z-direction (ft).
L_{zd} = dimensionless reservoir length in the z-direction.
m_l = slope of logarithmic derivative type curve.
m_{sq} = slope of square-root derivative type curve.
p = pressure (psi).
p_d = dimensionless pressure drop.
p_{dw} = dimensionless wellbore pressure drop.
q = sandface rate (bbl/day).
r_{xy} = square root of ratio k_x/k_y.
r_{zx} = square root of ratio k_z/k_x.
S_d = skin factor due to damage.
S_t = total skin factor due to damage and restricted entry.
t = time (hours).
t_d = dimensionless time.
x = distance along the reservoir x-axis (ft).
x' = x-coordinate of horizontal well axis (ft).
y = distance along the reservoir y-axis (ft).
y_1 = y-coordinate at the leading tip of the well (ft).
y_2 = y-coordinate at the ending tip of the well (ft).
y' = y-coordinate of any point on the horizontal well axis (ft).
y_j' = y-coordinate of a grid point on the horizontal well axis (ft).
z = distance along the reservoir z-axis (ft).
z' = z-coordinate of horizontal well axis (ft).

Greek symbols

Δp_{ly} = pressure drop in the y-direction due to an instantaneous line source at $y_1 < y' < y_2$ of unit strength (psi).

Δp_{sx} = pressure drop in the x-direction due to an instantaneous point source at $x = x'$ of unit strength (psi).

Δp_{sy} = pressure drop in the y-direction due to an instantaneous point source at $y = y'$ of unit strength (psi).

Δp_{sz} = pressure drop in the z-direction due to an instantaneous point source at $z = z'$ of unit strength (psi).

ϕ = reservoir porosity (fraction).

μ = fluid viscosity (cp).

π = constant (≈ 3.142).

Subscripts

d = dimensionless.

x = x-direction.

y = y-direction.

z = z-direction.

Appendix A: Mathematical model

Consider the following reservoir model. A well of radius r_w and length L_w is drilled parallel to the y-direction in a box-shaped reservoir. All the external boundaries are of no-flow type.

$$\frac{\partial p}{\partial \boldsymbol{n}} = 0, \quad \forall t, \tag{A.1}$$

where $\boldsymbol{n}$ is the vector perpendicular to the boundary planes.

The reservoir is initially at rest, that is,

$$p = p_i, \quad t = 0, \quad \forall x, y, z, \tag{A.2}$$

where the index i denotes initial conditions.

The reservoir dimensions are L_x, L_y and L_z, respectively. The well is located at x_0, z_0 and extends from $y_1 \leq y' \leq y_2$.

The reservoir is homogeneous and anisotropic. The well is produced at a constant rate q. The line source assumption is implied.

Fluid flow in a porous medium is governed by the diffusivity equation, which is given below.

$$\frac{k_x}{\mu}\frac{\partial^2 p}{\partial x^2} + \frac{k_y}{\mu}\frac{\partial^2 p}{\partial y^2} + \frac{k_z}{\mu}\frac{\partial^2 p}{\partial z^2} = \phi c \frac{\partial p}{\partial t}\,. \tag{A.3}$$

Appendix B: Dimensionless variables

Following Babu and Odeh [2,3], letting L_w denote the well length, we define the following dimensionless variables:

Dimensionless pressure drop $p_d(x_d, y_d, z_d, t_d)$:

$$p_d(x_d, y_d, z_d, t_d) = \frac{\sqrt{k_x k_y}\, L_z\, \Delta p(x,y,z,t)}{141.2 q\mu}\,. \tag{B.1}$$

Dimensionless time t_d:

$$t_d = \frac{4(0.000264)\sqrt{k_x k_y}\, t}{\phi c_t\, \mu L_w^2}\,. \tag{B.2}$$

Dimensionless position (x_d, y_d, z_d):

$$(x_d, y_d, z_d) = (2x/L_w, 2y/L_w, 2L_z/L_w). \tag{B.3}$$

Dimensionless reservoir dimensions $L_{xd} \times L_{yd} \times L_{zd}$:

$$L_{xd} \times L_{yd} \times L_{zd} = 2L_x/L_w \times 2L_y/L_w \times 2L_z/L_w. \tag{B.4}$$

$x-y$ permeability ration r_{xy}:

$$r_{xy} = \sqrt{\frac{k_x}{k_y}}\,. \tag{B.5}$$

$z-x$ permeability ratio r_{zx}:

$$r_{zx} = \sqrt{\frac{k_z}{k_x}}\,. \tag{B.6}$$

Appendix C: The mathematical solution

In terms of these dimensionless variables, the solution to eqs. (A.1), (A.2) and (A.3) is given by:

$$p_d(x_d, y_d, z_d, t_d) = \pi L_{zd} \int_0^{t_d} \frac{\Delta p_{sx}}{dx'_d} \left[\int_{y_{1d}}^{y_{2d}} \Delta p_{sy} \right] \frac{\Delta p_{sz}}{dz'_d} \, d\tau. \tag{C.1}$$

Here, we have assumed that the constant production rate q is uniformly distributed over the length of the well $L_w = y_2 - y_1$. This is the so-called uniform flux assumption.

A few general remarks on eq. (C.1) may be of interest.

The instantaneous "point" source solution for flow to the point (x', y', z') is given by the following [6]:

$$\Delta p_s(t) = \Delta p_{sx} \times \Delta p_{sy} \times \Delta p_{sz}, \tag{C.2}$$

where Δp denotes pressure drop and the point of interest is given by the coordinates x, y and z.

An instantaneous "line" source may be regarded as the superposition of an infinite series of point sources. Hence, the point source solution is integrated with respect to y_d along the axis of the well.

The continuous line source in turn may be regarded as a superposition of an infinite series of instantaneous line sources. Hence, the line source solution may be integrated with respect to τ from t_d to 0. As a result of these operations, we obtain eq. (C.1).

In terms of our dimensionless variables, the various components on the right-hand side of eq. (C.1) are given by the following:

$$\frac{\Delta p_{sx}}{dx'_d} = \frac{1}{L_{xd}} \left(1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi x_d}{L_{xd}} \cos \frac{n\pi x'_d}{L_{xd}} \exp\left(- \frac{n^2 \pi^2 r_{xy} t_d}{L_{xd}^2} \right) \right), \tag{C.3.1}$$

or equivalently,

$$\frac{\Delta p_{sx}}{dx'_d} = \frac{1}{2\sqrt{\pi r_{xy} t_d}} \sum_{n=-\infty}^{\infty} \sum_{i=1}^{2} \exp\left(- \frac{(x'_d + (-1)^i x_d - 2nL_{xd})^2}{4 r_{xy} t_d} \right). \tag{C.3.2}$$

Similarly,

$$\frac{\Delta p_{sz}}{dz'_d} = \frac{1}{L_{zd}} \left(1 - 2 \sum_{l=1}^{\infty} \cos \frac{l\pi z_d}{L_{zd}} \cos \frac{l\pi z'_d}{L_{zd}} \exp\left(- \frac{l^2 \pi^2 r_{xy} r_{zx}^2 t_d}{L_{zd}^2} \right) \right), \tag{C.4.1}$$

or equivalently,

$$\frac{\Delta p_{sz}}{dz'_d} = \frac{1}{2\sqrt{\pi r_{xy} r_{zx}^2 t_d}} \sum_{l=-\infty}^{\infty} \sum_{k=1}^{2} \exp\left(- \frac{(z'_d + (-1)^k z_d - 2lL_{zd})^2}{4 r_{xy} r_{zx}^2 t_d} \right). \tag{C.4.2}$$

Finally,

$$\left[\int_{y_{1d}}^{y_{2d}} \Delta p_{sy}\right] = \frac{(y_{2d}-y_{1d}}{L_{yd}} + \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\cos\left(\frac{m\pi(y_{2d}+y_{1d})}{2L_{yd}}\right)$$

$$\times \sin\left(\frac{m\pi(y_{2d}-y_{1d})}{2L_{yd}}\right)\exp\left(\frac{-m^2\pi^2(t_d/r_{xy})}{L_{yd}^2}\right), \qquad \text{(C.5.1)}$$

or,

$$\left[\int_{y_{1d}}^{y_{2d}} \Delta p_{sy}\right] = \frac{1}{2}\sum_{m=-\infty}^{\infty}\sum_{j=1}^{2}\left(\operatorname{erf}\left(\frac{(y_{2d}+(-1)^j y_d - 2mL_{yd})}{2\sqrt{t_d/r_{xy}}}\right)\right.$$

$$\left.-\operatorname{erf}\left(\frac{(y_{1d}+(-1)^j y_d - 2mL_{yd})}{2\sqrt{t_d/r_{xy}}}\right)\right). \qquad \text{(C.5.2)}$$

The calculation procedure for efficient evaluation of the above infinite series has been described in a companion paper [7].

Appendix D: Limiting forms of the analytic solution

Early radial drawdown:

The drawdown equation in this period is given by:

$$p_{wf} = p_i - \frac{162.6q\mu B}{\sqrt{k_x k_z}\, L_w}\left(\log\frac{\sqrt{k_x k_z}\, t}{\phi\mu c_t r_w^2} - 3.23 + 0.87 S_d\right). \qquad \text{(D.1)}$$

Early linear drawdown:

In the early linear flow period, the flow behaviour is given by:

$$p_w = p_i - \frac{8.13q\mu B}{L_w h}\left[\sqrt{\frac{t}{\phi\mu c_t k_x}}\right.$$

$$\left. + \frac{17.37h}{\sqrt{k_x k_z}}\left(\ln\frac{h}{r_w} + 0.25\ln\frac{k_x}{k_z} - \ln\left(\sin\frac{\pi z_0}{h}\right) - 1.838 + S_d\right)\right]. \qquad \text{(D.2)}$$

Late radial drawdown:

The drawdown equation for this period is given by:

$$p_{wf} = p_i - \frac{162.6 q\mu B}{\sqrt{k_x k_y}\, h}\left[\log\frac{k_y t}{\phi\mu c_t L_w^2} - 1.76 + 0.87\sqrt{\frac{k_y}{k_z}}\frac{h}{L_w}\left(\ln\frac{h}{r_w} + 0.25\ln\frac{k_x}{k_z} - \ln\left(\sin\frac{\pi z_0}{h}\right) - 1.838 + S_d\right)\right]. \quad (D.3)$$

Late linear drawdown:

In the late linear flow period, the drawdown behaviour is given by:

$$p_w = p_i - \frac{8.13 q\mu B}{L_y h}\left[\sqrt{\frac{t}{\phi\mu c_t k_x}} + \frac{17.37h}{\sqrt{k_x k_z}}\left(\ln\frac{h}{r_w} + 0.25\ln\frac{k_x}{k_z} - \ln\left(\sin\frac{\pi z_0}{h}\right) - 1838 + S_t\right)\right]. \quad (D.4)$$

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